UNIVERSITY OF
KWAZULU-NATAL
INYUVESI
YAKWAZULU-NATALI

SCHOOL OF ENGINEERING

MAIN EXAMINATIONS: NOVEMBER 2019

COURSE AND CODE: PROCESS MODELLING & OPTIMISATION ENCH3PO

DURATION: 3 HOURS

TOTAL MARKS: 100

INTERNAL EXAMINER(S): DR S MOODLEY
PROF I GOVENDER

INTERNAL MODERATOR(S): DR K MOODLEY

EXTERNAL MODERATOR: PROF BF BAKARE

INSTRUCTIONS:

1. **ALL** questions should be attempted.

2. **Two answer books are provided. Label them Book 1 and Book 2 respectively. Answer Section A in Book 1 and Section B in Book 2.**

3. Any programmable or non-programmable calculator may be used provided it has been cleared of any information that would subvert the purpose of the examination.

4. Calculations must be shown in sufficient detail to illustrate your understanding of the procedure.

5. This examination question paper, together with all associated diagrams **MUST BE HANDED IN TOGETHER WITH YOUR SCRIPT.**

SECTION A

QUESTION 1

A consultant to a medical institution was asked to determine the physiological heat released from an athlete's forearm. The forearm can be modelled as a cylinder with radius of 6 cm and a length of 60 cm (a very big athlete!). The ambient temperature is T_a = 25°C and the temperature at the centre of the forearm is T_c = 37°C (body temperature). Because of the cooling effect of the surrounding air, the forearm must generate heat to maintain the temperature at the centre of the forearm at 37°C. The thermal conductivity of the forearm is k = 0.7 W/m K. There is a stagnant film of air surrounding the arm, and the heat transfer through this film is characterised by the heat transfer coefficient h (heat flux through the stagnant film is $q = h\Delta T$, where ΔT is the temperature driving force across the film).

1.1) Derive the steady-state heat balance equation for the forearm and show that it has the form

$$\frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + Q = 0$$

where Q is the volumetric heat production rate. Carefully state the boundary conditions.

(8)

1.2) Show that the temperature distribution takes the form

$$T = T_a + \frac{QR^2}{4k}\left[1 - \left(\frac{r}{R}\right)^2\right] + \frac{QR}{2h}$$

(9)

1.3) Show that the heat per unit volume that the forearm generates per unit time is

$$Q = \frac{T_c - T_a}{\left[\left(\frac{R^2}{4k}\right) + \left(\frac{R}{2h}\right)\right]}$$

and hence calculate Q if h = 15 W/m^2/K.

(4)

1.4) This athlete is competing in the winter games where the air is –10°C. If the forearm of this athlete generates the same amount of heat as determined in (1.3), calculate the skin temperature and the temperature at the centre of the forearm for the same heat transfer coefficient as in (1.3).

(4)

TOTAL /25/

QUESTION 2

A new lotion is being developed at a well-known cosmetic company called 3PO. The engineers at this company are conducting tests to ensure that this lotion can absorb and diffuse into the skin within a reasonable time. The principal task is to develop a scheme to measure the diffusivity of the lotion as it courses through the skin. Here is the description of the problem:

(i) The skin is very thin so you can ignore curvature effects and the thickness of the skin is taken to be L.

(ii) The initial thickness of a very thin film of lotion above the skin is $\delta_0 = \delta(t = 0)$; t is time in seconds.

(iii) The lotion dissolves into the skin with a constant solubility, C_0, and the concentration units can be taken as moles of lotion per unit volume of the skin.

(iv) The diffusion of the lotion within the skin depends on the concentration and the flux equation takes the form $J = -D\frac{dC}{dx}$. Here you can assume that the diffusivity is a linear function of concentration,

$$D = D_0(1 + \alpha C)$$

where D_0 and α are constants.

(v) You may assume that the lotion concentration at the bottom of the skin layer is negligible, and the total layer of skin is initially void of lotion.

(vi) Denote the depth below the skin-lotion interface by x, i.e. the origin of your coordinate system is at the skin-lotion interface with the positive x-direction pointing vertically down into the skin.

2.1) Show that the mass balance equation to describe the distribution of lotion in the skin is (you may assume a quasi-steady state for the transport of lotion in the skin):

$$\frac{d}{dx}\left[D_0(1 + \alpha C)\frac{dC}{dx}\right] = 0$$

(5)

2.2) Show that the solution of the mass balance equation is

$$1 - \frac{(1 + \alpha C)^2 - 1}{(1 + \alpha C_0)^2 - 1} = \frac{x}{L}$$

(8)

2.3) Show that the molar flux of lotion into the skin is

$$J = \frac{D_0}{2\alpha L}[(1 + \alpha C_0)^2 - 1]$$

(5)

QUESTION 2 continued ...

2.4) To determine how fast the lotion disappears from the top; you need to set up a transient mass balance around the lotion film layer. Derive this first order differential equation if ρ_L is the molar density of lotion.

(7)

TOTAL /25/

SECTION B

QUESTION 3

An experiment was conducted to investigate the effect of temperature on the dynamic viscosity of a liquid. The results are tabulated below:

T (K)	μ (Pa.s)
273	2×10^{-3}
298	1×10^{-3}
323	6×10^{-4}

It is proposed to fit the data to a viscosity model of the form

$$\mu = A \times 10^{B/(T-140)}$$

where A and B are constants (i.e. the model's parameters), μ has units of Pa.s, and T has units of Kelvin. Use the Gauss-Newton method for nonlinear least squares regression to determine the values of the model parameters, and state the units of each parameter. A suitable set of initial values for A and B are 2×10^{-5} and 245, respectively. Perform one iteration only.

Supporting Information:

$$Z^{\mathrm{T}} Z \Delta b = Z^{\mathrm{T}} D$$

TOTAL /15/

QUESTION 4

Consider a solid sphere of radius R, emissivity ε and thermal conductivity k. The sphere is giving off heat to its surroundings (which is at a constant temperature T_∞) via radiation, at a known radiative flux q''_{rad}. Within the sphere, heat is generated at a uniform volumetric rate $\dot{q_V}$ (in W/m^3).

4.1) Show that the ordinary differential equation that describes the steady-state radial temperature distribution within the sphere is

$$\frac{\mathrm{d}^2T}{\mathrm{d}r^2}+\frac{2}{r}\frac{\mathrm{d}T}{\mathrm{d}r}+\frac{\dot{q_V}}{k}=0$$

Also, write down the boundary conditions.

(12)

4.2) Hence, by using two equally-spaced nodes between $r = 0$ and $r = R$, calculate the temperature distribution within the sphere by using a finite difference scheme, given the following data:

Property or variable	Value	Unit(s)
R	0.3	m
ε	0.95	dimensionless
k	0.2	W.m^{-1}.K^{-1}
T_∞	300	K
q''_{rad}	100	W.m^{-2}
$\dot{q_V}$	1000	W.m^{-3}
σ	5.67×10^{-8}	W.m^{-2}.K^{-4}

(13)

Supporting information:

$$y''\approx(y_{i-1}-2y_i+y_{i+1})/(\Delta x)^2$$

$$y'\approx(y_{i+1}-y_{i-1})/(2\Delta x)$$

$$q_{\text{rad}}=\varepsilon\sigma A(T^4-T_\infty^4)$$

TOTAL /25/

QUESTION 5

The partial differential equation that describes the spatially transient temperature distribution $T(x,t)$ in a one-dimensional rod of length L, which loses heat to its surroundings via convection, is

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} - \beta(T - T_\infty)$$

In the preceding equation, α, β and T_∞ are constants. The rod is initially at a uniform temperature T_R. Then, at $t = 0$, its left end ($x = 0$) is subjected to a constant temperature T_0 while its right end ($x = L$) is maintained at a constant temperature T_L. Using a centred-difference approximation for the spatial derivative and a forward-difference approximation for the temporal (time) derivative, devise a *simple* *implicit* finite difference scheme that can be used to solve for the temperatures at three equally spaced interior nodes in the rod, after the *first* time-step. Your final answer must be in symbolic matrix-vector format i.e.

$$[A]\underline{T} = \underline{b}$$

where $[A]$ is a coefficient matrix, $\underline{T}$ is the vector of unknown temperatures and $\underline{b}$ is a vector of constants.

TOTAL /10/